

Phase Transitions
between
Hadronic and Partonic Worlds

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Outline:

- ◆ What T do we see in $A+A$ and elementary particle reactions? Do these T signal PT?
- ◆ A bit of history: Stat. Bootstrap Model; MIT Bag Model... Problems with GCE.
- ◆ MCE: Properties of Hagedorn resonances = Perfect Thermostats and Particle Reservoirs
- ◆ Open Questions and Conclusions

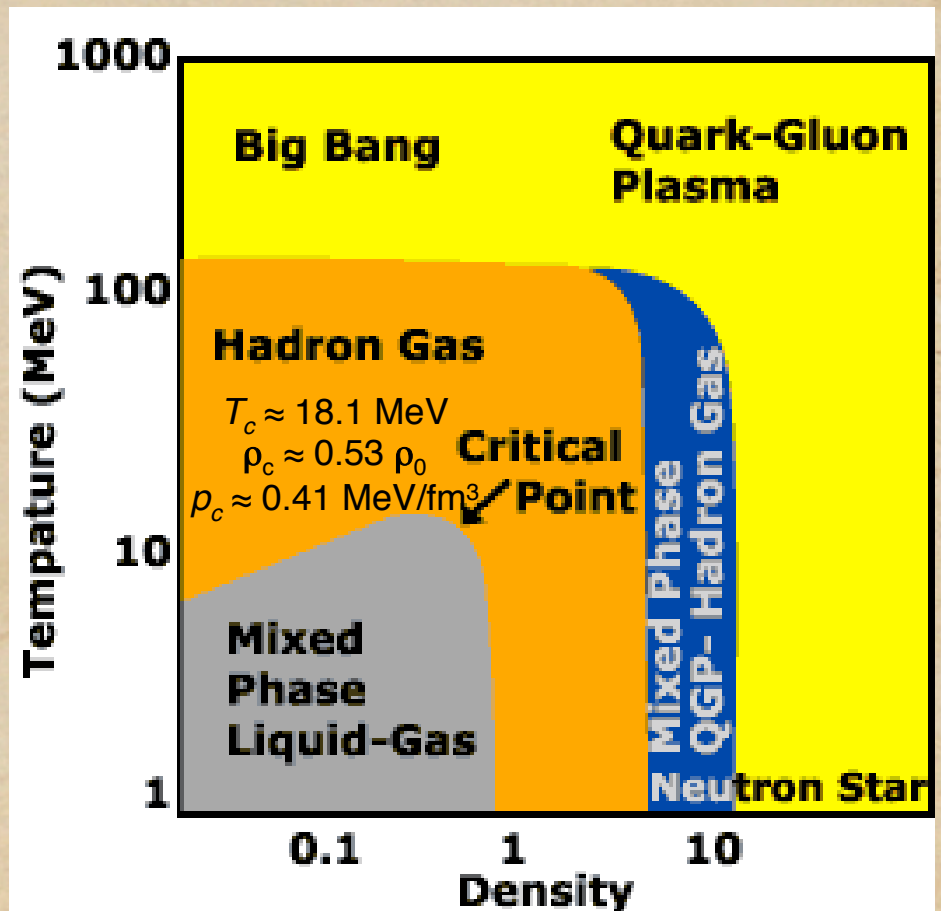
Phase Diagram

◆ Partonic World:

\Leftrightarrow Hadronic World

Transition:

This Talk



Temperatures in A+A Reactions

- ◆ **Lattice QCD** at 0 baryonic density:

transition $T = 170 \pm 10 \text{ MeV}$, F.Karsch,
Nucl.Phys.Proc.Suppl. 83(2000)

- ◆ **Chemical Freeze-out** at highest SPS and all RHIC energies $T = 170 \pm 10 \text{ MeV}$:

G.D.Yen, M.I.Gorenstein, PRC 59 (1999), P.Braun-Munzinger et al PLB 465 (1999)

This T shows that hadronic composition of a created matter (including decay of resonances!) does not change while system expands and cools down.

Remarkably, $T = \text{Const}$ while \sqrt{s} grows by 12 times!

Early Hadronization Temperature in A+A Collisions

- ◆ Remarkably, at highest SPS and all RHIC energies it is also $T = 170 \pm 10$ MeV!
- ◆ This T evidences that momentum spectra of some hadrons are frozen since the moment of their formation!
- ◆ Necessary conditions: heavy hadrons, small cross-sections with other hadrons, no low lying resonances with pions!

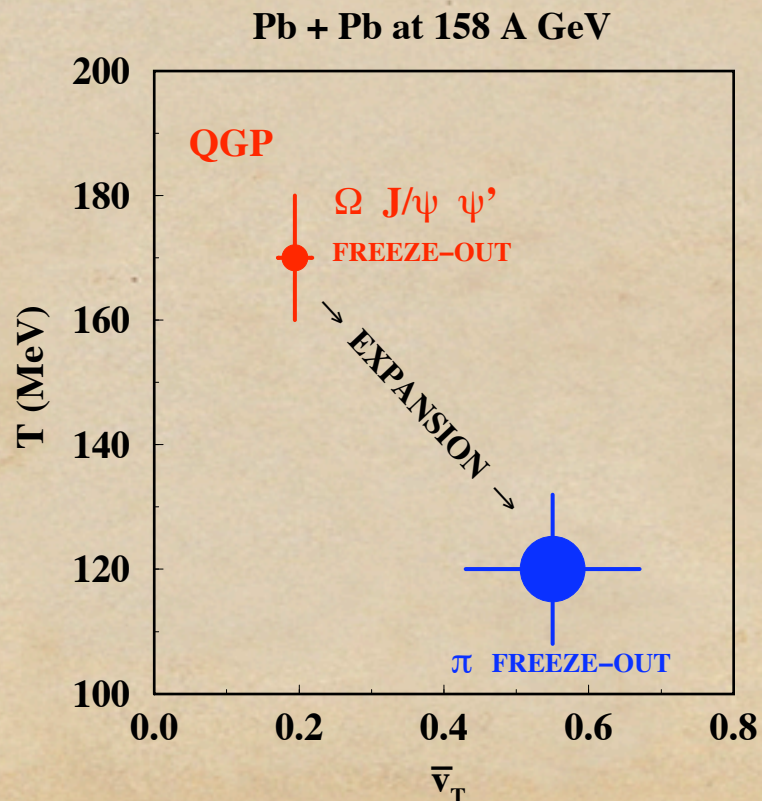
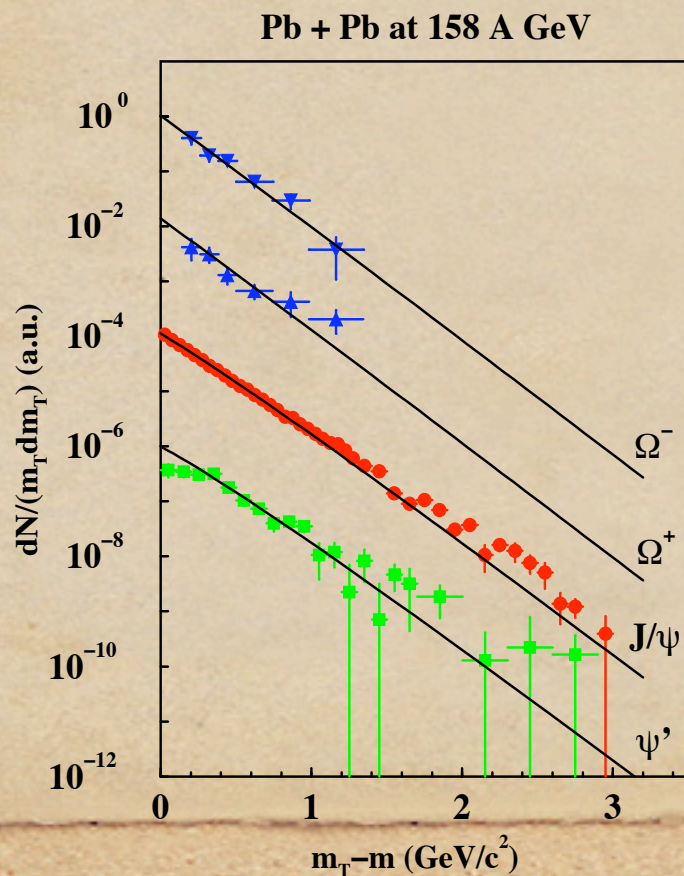
Momentum Spectra at CERN SPS

- ◆ Ω J/ψ ψ' transverse momentum spectra indicate: $T = 170 \pm 10$ MeV

Is their hadronization T!

- ◆ An elaborate Blast Wave approximation was used to fit data

M.I. Gorenstein, K.A.B., M. Gaździcki, Phys. Rev. Lett. **88** (2002) 1323011

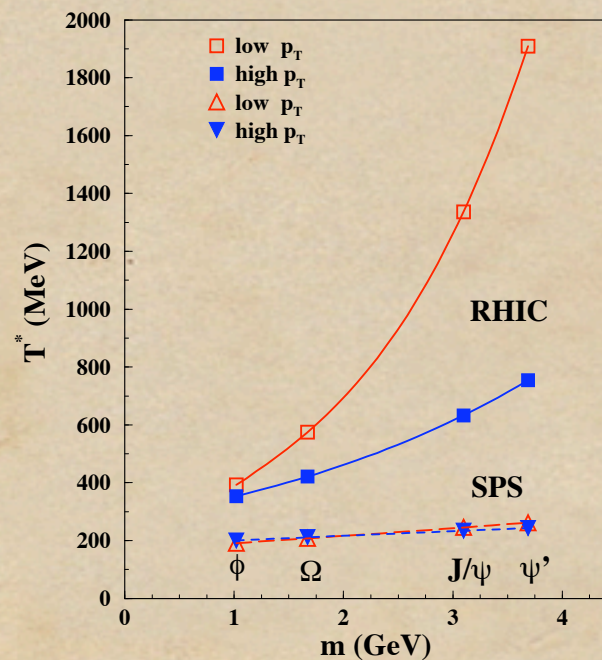
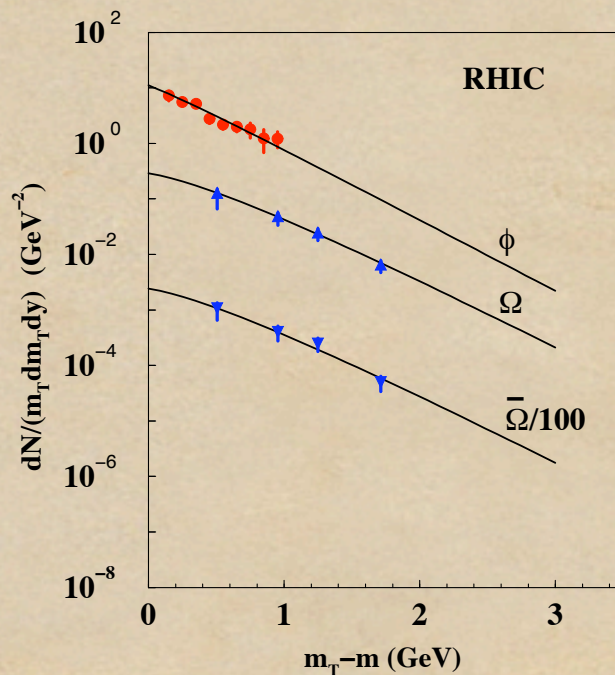


m_T -Spectra at RHIC $\sqrt{s}_{NN} = 130\text{GeV}$

- ◆ ϕ, Ω transverse momentum spectra and emission volume $\tau_H R_H^2$ show: $T = 170 \pm 5 \text{ MeV}$ is their hadronization T!

K.A.B., M. Gazdzicki, M.I. Gorenstein, PRC **68** (2003): $\chi^2/ndf \cong 0.46$

$\lambda_{\Omega^-} = 1.09 \pm 0.06$, $y_T^{max} = 0.74 \pm 0.09$, $\tau_H R_H^2 = 275 \pm 70 \text{ fm}^3/c$



ϕ data: STAR, Phys. Rev. **C 65** 041901(R) (2002) ;

Ω^\pm data: G. van Buren [STAR] , talk at QM2002 .

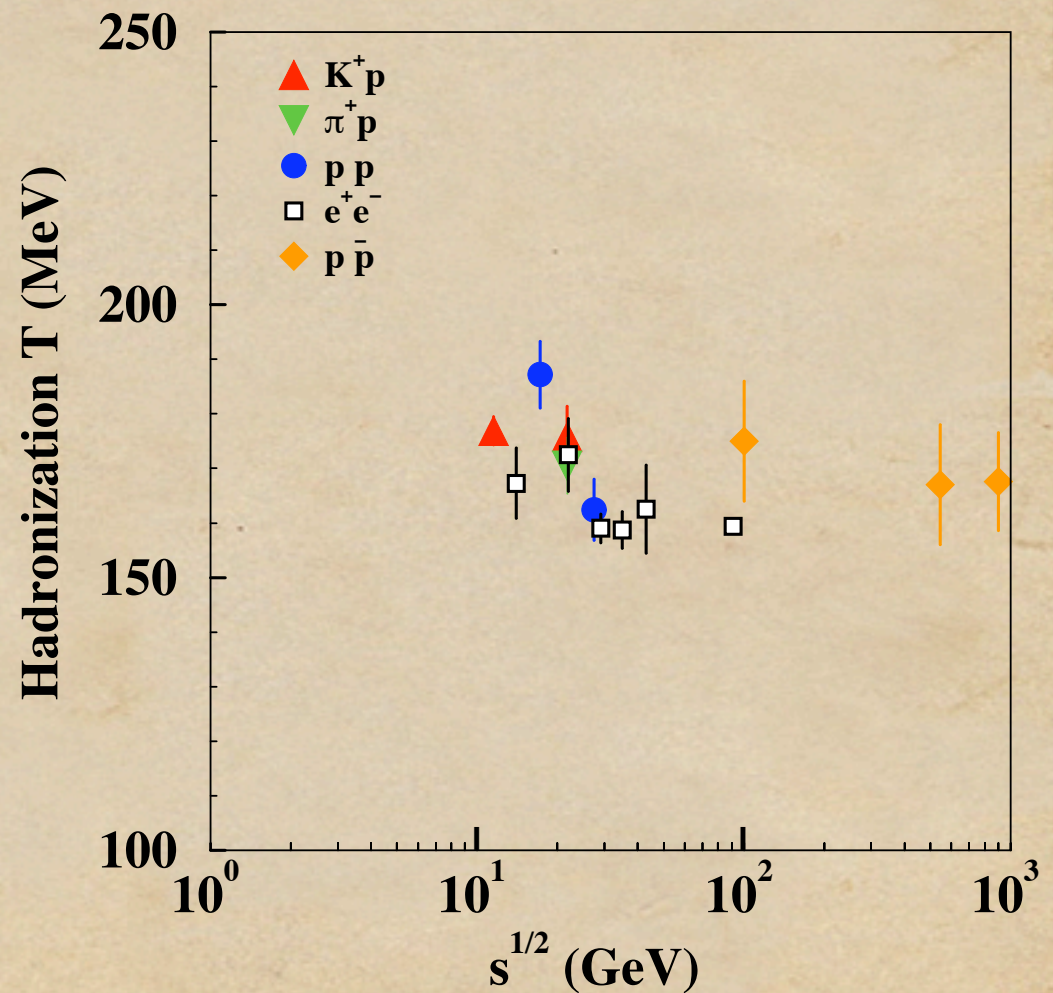
Evident Explanation for A+A reactions

- ◆ For 0 baryonic charge the Particle Ratios (chemical freeze-out) are frozen since hadronization of QGP at $T \approx 170 \pm 10$ MeV.
- ◆ For 0 baryonic charge the kinetic freeze out of some hadrons (ϕ , J/ψ , ψ' mesons, Ω hyperons) occurs at their hadronization from QGP at same T!
- ◆ Surprisingly, similar values of T are seen in el. particle collisions!

Hadronization in Elementary Particle Collisions

◆ Stat. Hadronization
Model: $T = 175 \pm 15$ MeV

F. Becattini, A. Ferroni, Acta.
Phys. Polon. B 35 (2004)



Kaon Inverse Slopes in Elementary Particle Collisions

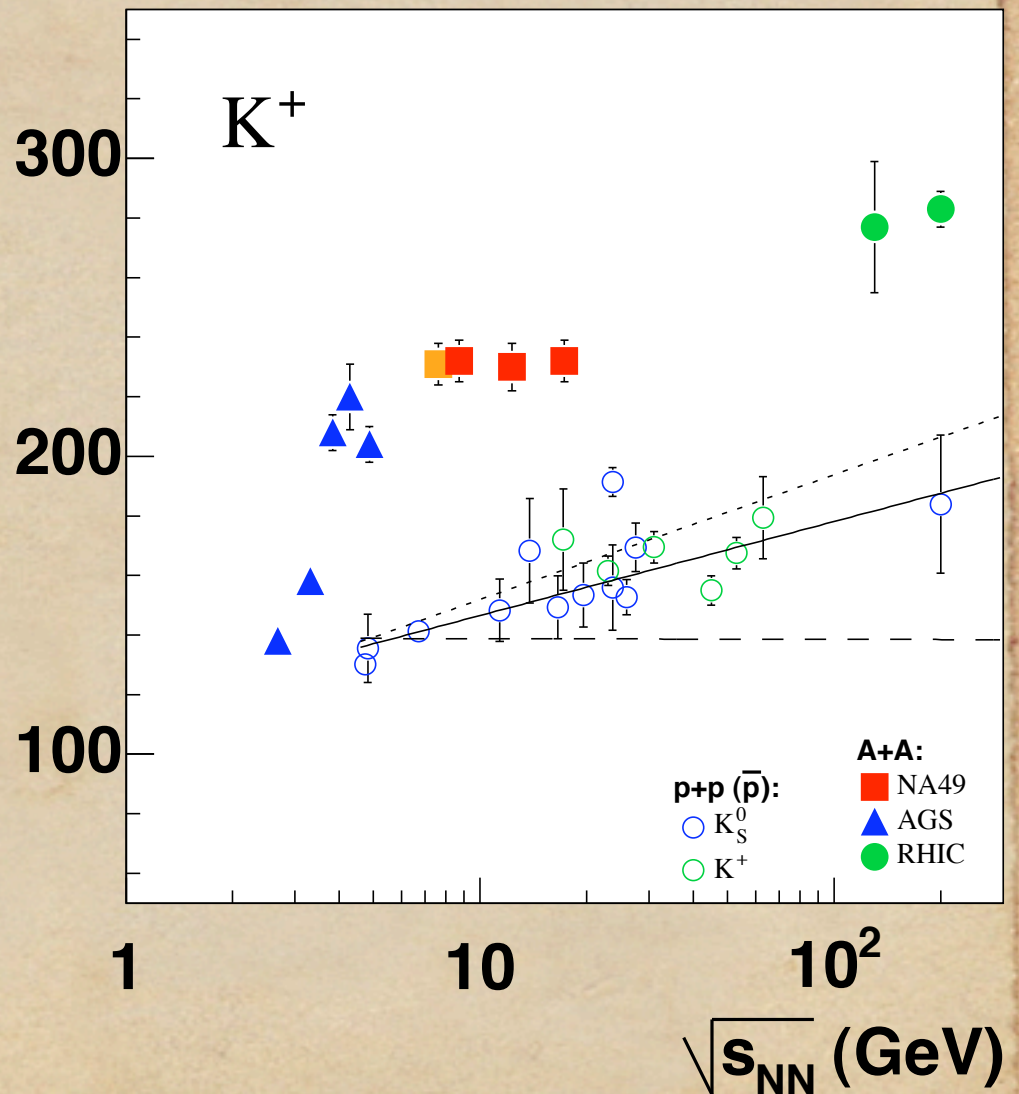
- ◆ In wide \sqrt{s} range
 $T = 180 \pm 20 \text{ MeV}$, $T^*(\text{MeV})$

M.Kliemant, B.Lungwitz, M.Gazdzicki,
 PRC 69 (2004) (Open symbols)

- ◆ About the same T is for
 pions and nucleons

- ◆ A+A data is a Step,
 inverse slopes are modified
 due to transverse expansion

M.Gorenstein, M.Gazdzicki, K.A.B.,
 PLB 567 (2003)



Problem

- ◆ Same $T = 170 \pm 10$ MeV values in A+A collisions are explained by transition to/from QGP.
- ◆ Why T values in El. Particle collisions are nearly the same? Is there QGP formed? Why don't we see it?
- ◆ Do Const T values in El. Particle collisions signal a phase transition?
- ◆ Usually it depends on conditions:
pressure = Const, or Volume = Const, or ...

There is gap in our understanding of
 $A+A$ and $h+h$ reactions!

$A+A$ data

**$h+h$, $e+e$
data**



A photograph of ancient stone ruins, likely a wall or foundation, under a clear blue sky with scattered clouds. In the background, there are green trees and distant mountains. A pink rectangular box with a light blue border is superimposed over the middle of the image, containing the text "Statistical Bootstrap Model".

Statistical Bootstrap Model

Statistical Bootstrap Model

The first evidence for $\rho(E) = C e^{\alpha E}$ density of states was found **numerically** in 1958 having 15 particles only!

Result was not understood until a model was formulated in 1963

Model predicted: entropy $S = \alpha E$ energy

G. Fast and R. Hagedorn, Nuovo Cimento **27** (1963) 208

Then: $T = 1/\alpha = \text{Const}$ leads to the following density of states:

$\rho(E) = C e^S = C e^{\alpha E}$ i.e. **exponentially growing spectrum!**

R. Hagedorn, Suppl. Nuovo Cimento **3** (1965) 147

- ◆ It was *heresy* and Weisskopf forbade to publish it as CERN preprint! But 1964 data confirmed an exponential form.

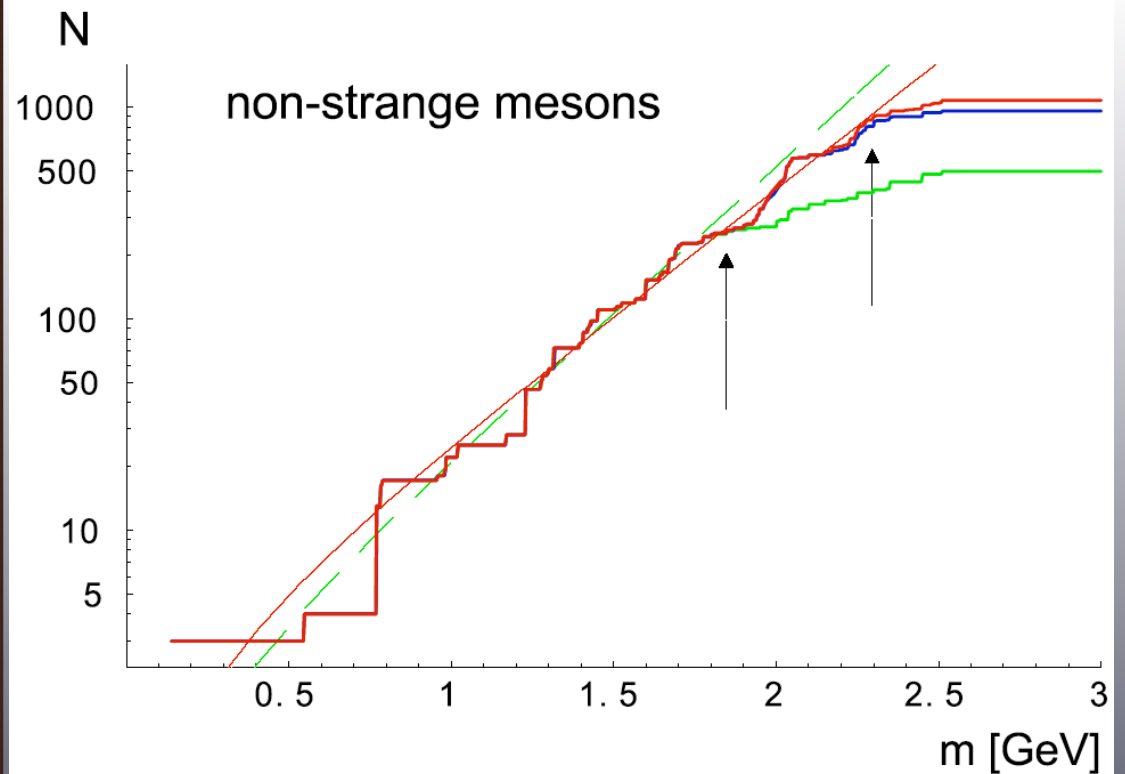
Hagedron Spectrum Follows from

$$\rho(m) \approx m^{-3} \exp \left[\frac{m}{T_H} \right] \text{ for } m \rightarrow \infty$$

Stat.Bootstrap Model,
S.Frautschi, 1971

Veneziano Model,
K.Huang,S.Weinberg,
1970

M.I.T. Bag Model,
J.Kapusta, 1981



$$N_{\text{exp}}(m) = \sum_i g_i \Theta(m - m_i(\text{GeV}))$$

Hagedron Spectrum and Bag Model

- ◆ Bag Model is a foundation of our phenomenology. It gave a first evidence that transition to Partonic World is a phase transition. Resonances are small bags of QGP.
- ◆ How does it explain Hagedorn spectrum with Const T?
- ◆ Consider a single heavy bag of 0 baryonic charge in vacuum. 0 external pressure fixes the temperature (g is # d.o.f):

$$p = g \frac{\pi^2}{90} T_H^4 - B = 0 \quad \Rightarrow \quad T_H = \left[\frac{90}{g\pi^2} B \right]^{\frac{1}{4}}$$

Then entropy of the bag is

$$S = \frac{\varepsilon(T_H)V}{T_H} \equiv \frac{Mass}{T_H} \quad \Rightarrow \quad \rho(Mass) = \exp[S] = \exp \left[\frac{Mass}{T_H} \right]$$

Everything looks fine, BUT...

Example #1: 1-d Harmonic Oscillator

- ◆ For 1-d Harmonic Oscillator with energy ϵ in contact with Hagedorn resonance (just exponential spectrum ρ_H for simplicity). Total energy is E .

- ◆ The microcanonical probability of state ϵ is:

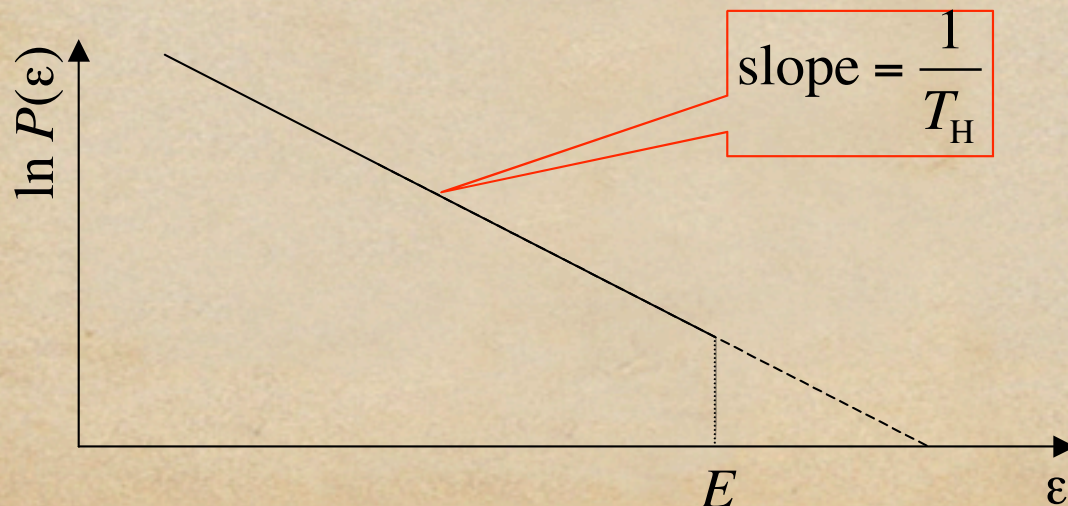
$$P(\epsilon) = \rho(E - \epsilon) = \exp\left(\frac{E - \epsilon}{T_H}\right) = \exp\left(\frac{E}{T_H}\right) \exp\left(-\frac{\epsilon}{T_H}\right)$$

Exponent is
Grand canonical!
With fixed T !

Average value of ϵ is

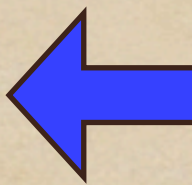
$$\bar{\epsilon} = T_H \left(1 - \frac{E/T_H}{\exp(E/T_H) - 1} \right)$$

For $E \rightarrow \infty$: $\bar{\epsilon} \rightarrow T_H$



Example #2: An Ideal Vapor coupled to Hagedorn resonance

- ◆ Consider microcanonical partition of N particles of mass m and kin. energy ε . The total level density is

$$P(E, \varepsilon) = \rho_H(E - \varepsilon) \rho_{iv}(\varepsilon) = \frac{V^N}{N! \left(\frac{3}{2}N\right)!} \left(\frac{m\varepsilon}{2\pi}\right)^{\frac{3}{2}N} \exp\left(\frac{E - mN - \varepsilon}{T_H}\right)$$


Exponent is
Grand canonical!
With fixed T !

The most probable energy partition is

$$\frac{\partial \ln P}{\partial \varepsilon} = \frac{3N}{2\varepsilon} - \frac{1}{T_H} = 0 \Rightarrow \frac{\varepsilon}{N} = \frac{3}{2}T_H$$

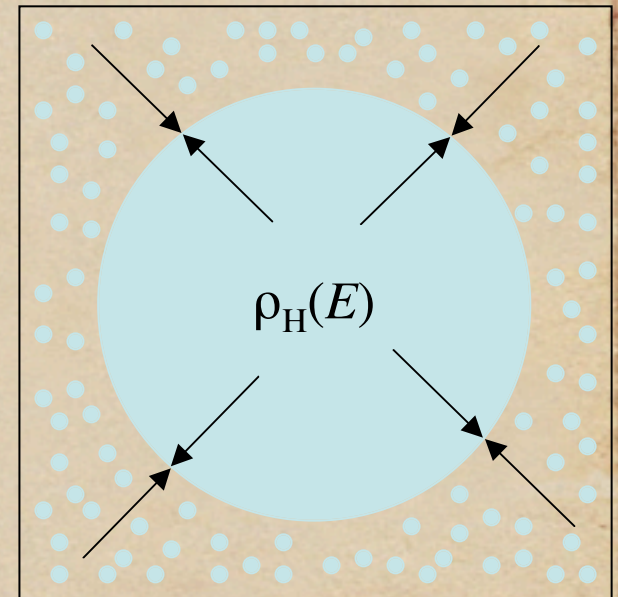
- T_H is the sole temperature characterizing the system:
- **A Hagedorn-like system is a perfect thermostat!**

Intermediate Conclusion:

- ◆ For Hagedorn resonances the Grand canonical ensemble with T other than Hagedorn T does not make any sense!
- ◆ Because it is equivalent to bring in contact 2 thermostats with different T

Example #3: An Ideal Particle Reservoir

- ◆ If, in addition, particles are generated by the Hagedorn resonance, their concentration is **volume independent!**



$$\left. \frac{\partial \ln P}{\partial N} \right|_V = -\frac{m}{T_H} + \ln \left[\frac{V}{N} \left(\frac{m T_H}{2\pi} \right)^{\frac{3}{2}} \right] = 0 \Rightarrow \frac{N}{V} = \left(\frac{m T_H}{2\pi} \right)^{\frac{3}{2}} \exp \left(-\frac{m}{T_H} \right)$$

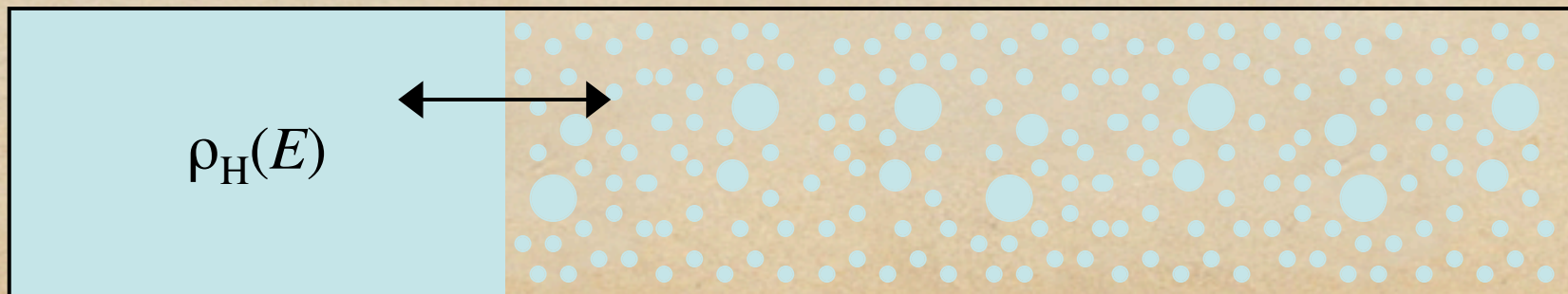
ideal vapor ρ_{iv}

- particle mass = m
- volume = V
- particle number = N
- energy = ε

Remarkable result because it mean saturation between gas of particles and Hagedorn thermostat!

Important Finding!

- ◆ Volume independent concentration of vapor means:
- ◆ for increasing volume of system gas particles will be evaporated from Hagedorn resonance (till it vanishes);
- ◆ by decreasing volume we will absorb gas particles to Hagedorn resonance! Compare to ordinary water!
- ◆ Literally, it is a liquid (Hagedorn resonance) in equilibrium with its vapor!



The Story so far...

- ◆ Anything in contact with a Hagedorn thermostat acquires the Hagedorn temperature.
- ◆ If particles (e.g. pions) can be created from a Hagedorn thermostat, they will form a saturated vapor at fixed (Hagedorn) temperature.
- ◆ If different particles (i.e. of different masses m) are created, they will be in chemical equilibrium.
- ◆ Because of these properties the radiant Hagedorn resonance should be similar to a compound nucleus (same spectra and branching ratios), but at fixed T .

The role of the lower mass cut-off

- ◆ So far we ignored that for light hadrons the spectrum is not exponential. Also translational d.o.f. of the Hagedorn thermostat were ignored.

- ◆ For a single Hagedorn thermostat ($a = \text{const}$):

$$\rho_H(m_H) = \exp[m_H/T_H](m_0/m_H)^a \text{ for } m_H \geq m_0$$

The mass cut-off: $m_0 \gg T_H$

From an analysis by W. Broniowski et. al., hep-ph/0407290 \Rightarrow
 $m_0 < 2 \text{ GeV}$.

For a single Hagedorn thermostat:

$$\frac{\delta \ln P}{\delta m_H} = \frac{1}{T_H} + \left(\frac{3}{2} - a\right) \frac{1}{m_H^*} - \frac{3(N+1)}{2 E_{kin}} = 0$$

$$T^*(m_H^*) \equiv \frac{2 E_{kin}}{3(N+1)} = \frac{T_H}{1 + \left(\frac{3}{2} - a\right) \frac{T_H}{m_H^*}}.$$

N-dependence and Kinematic Limit

- ◆ For such N the maximum of microcanonical partition exists.

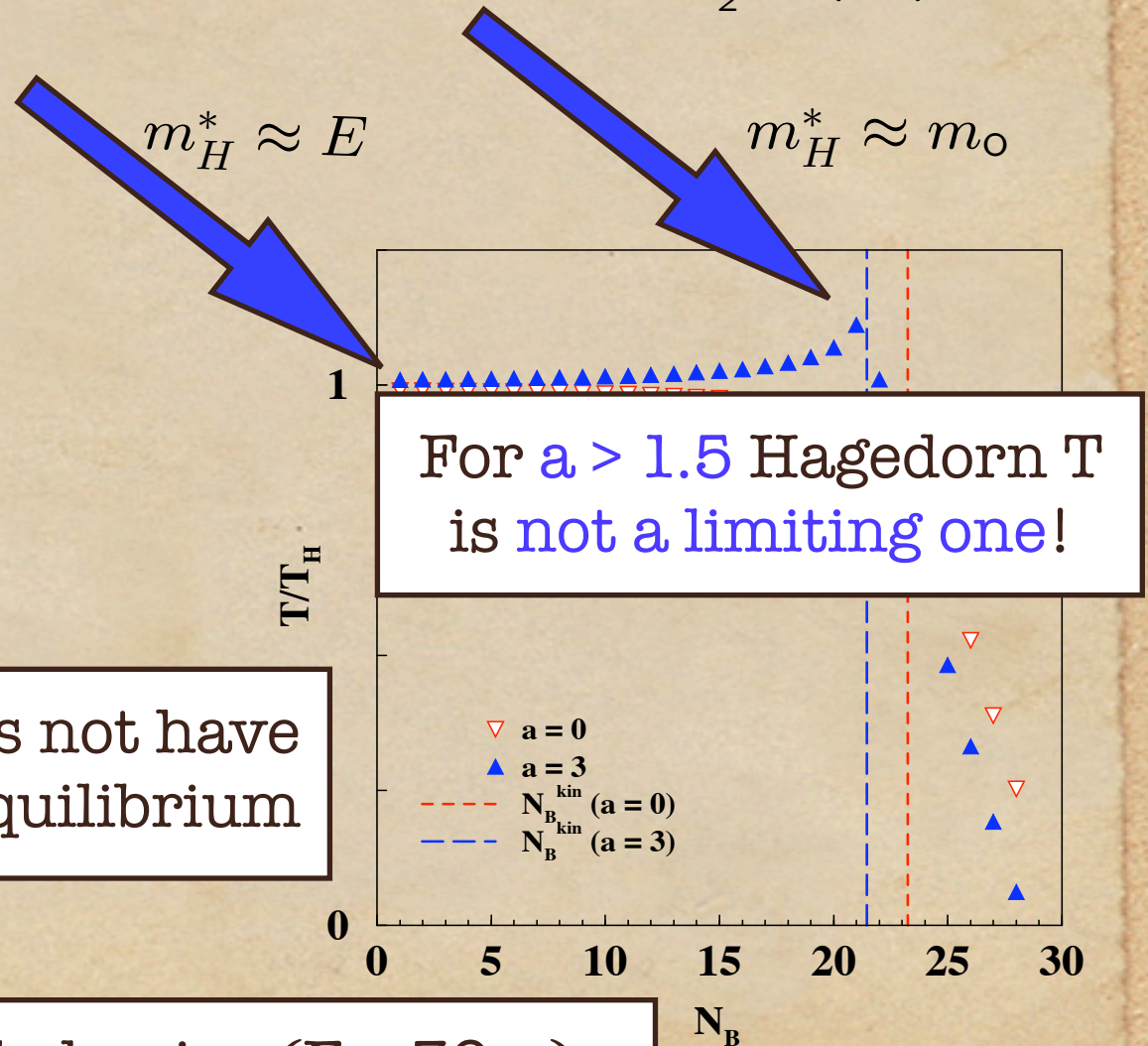
$$N \leq N_B^{kin} \equiv \frac{E - \left[\frac{m_o}{T_H} - a\right] T^*(m_o)}{m + \frac{3}{2} T^*(m_o)}.$$

- ◆ Otherwise, for

$$N > N_B^{kin}, \Rightarrow$$

$$T = T_o(N) \equiv \frac{2(E - mN - m_o)}{3(N+1)}.$$

Hagedorn resonance does not have sufficient mass to keep equilibrium



A typical behavior ($E = 30m$)

Inverse Slopes

- ◆ The microcanonical partition can be cast

$$\text{For } N \leq N_B^{kin} \Rightarrow P = V \rho_H(m_H^+) \int \frac{d^3 Q}{(2\pi)^3} e^{-\frac{\sqrt{m_H^{+2} + Q^2}}{T^*(m_H^+)}} \\ \frac{e^{\frac{E}{T^*(m_H^+)}}}{N!} \left[V g \int \frac{d^3 p}{(2\pi)^3} e^{-\frac{\sqrt{m^2 + p^2}}{T^*(m_H^+)}} \right]^N .$$

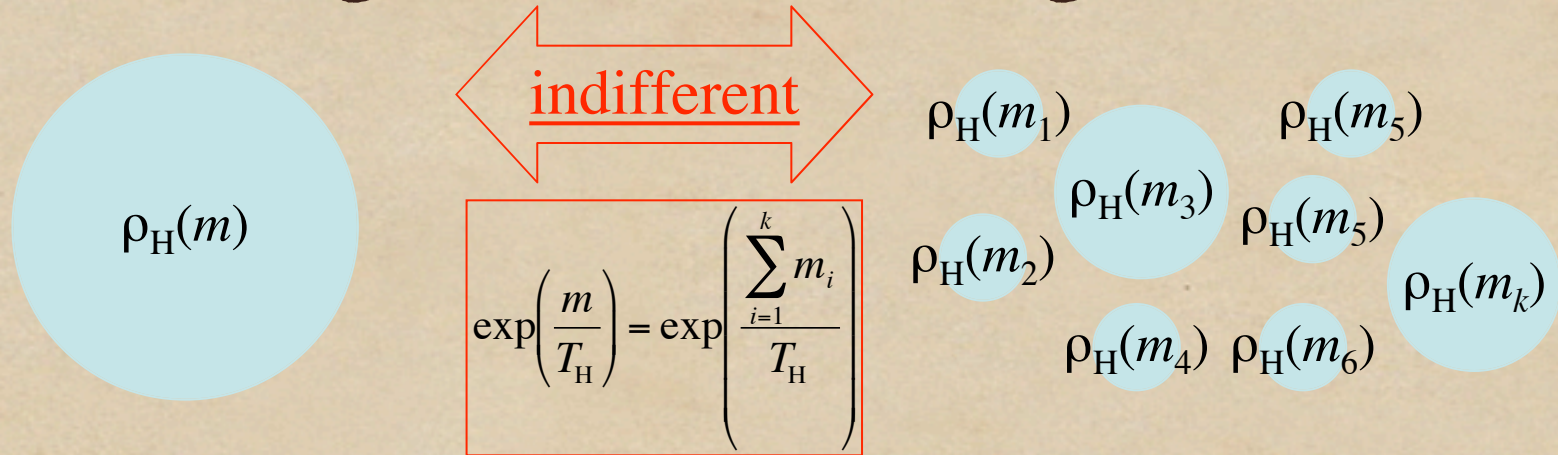
For $N > N_B^{kin}$ one has to replace $T^*(m_H^+) \leftarrow T_o(N)$ and $m_H^+ \leftarrow m_o$

Inverse slope of momentum distribution
is a temperature!

- Lower mass cut-off does not affect our results much.
- In N_B^{kin} vicinity there may exist 10–20 % effect on T^*

Stability Against Fragmentation

For no translational entropy the Hagedorn thermostat (=bag) is **indifferent** to fragmentation.



Translational d.o.f. do not change this result.

Present model **not only EXPLAINS** why Becattini's

Stat. Hadronization Model gives T **close to Hagedorn T** ,

but it also **justifies** the validity of his major assumption that

all fireballs originate from a **Singe Protofireball!**

How to observe it?

- ◆ In vacuum a Hagedorn thermostat radiates hadrons. For slow radiation the pressure due to radiation is small (2 - 3 % of Bag pressure). Thus, measuring **energy and volume** (HBT) for vanishing baryon number, one can find the # of d.o.f. **g** :

$$\varepsilon = g \frac{\pi^2}{30} T_H^4 + B$$

$$p = g \frac{\pi^2}{90} T_H^4 - B \approx 0$$

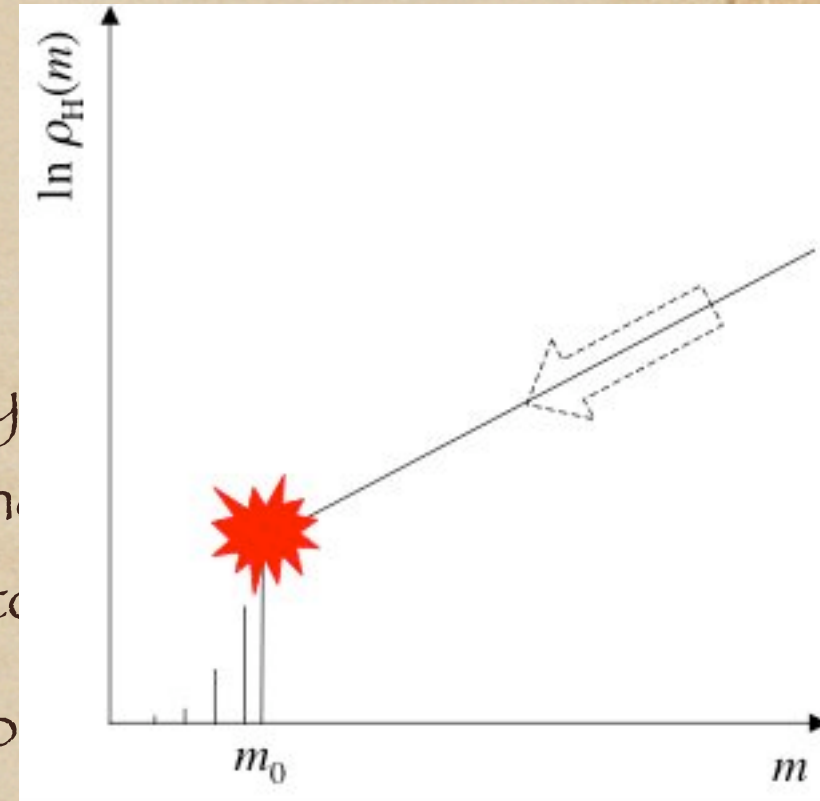
$\rho_H(E)$

Gev/fm³

There was an attempt by Purdue group to measure energy density in el. particle collisions.
See T.Alexopoulos et al, PLB 528 (2002)

Flash at Double Phi Decay?

- ◆ Different models show that parameter a in Hagedorn mass spectrum is $a=3$ or even $a>3$.
- ◆ In this case at the end of radiation
- ◆ $m_H^* \rightarrow m_0$ and $T^* \approx 1.1 T_H - 1.2 T_H$
- ◆ For pions it is unobservable, but for heavy
Thus, **heavy hadrons** emitted about the end
have an enhanced probability, compared to
- ◆ Best candidate to see **Flash** (V. Koch) is, p
decay?
- ◆ For more definite predictions we need **better model and better data!**



Conclusions

- ◆ Exponential mass spectrum is a very special object.
- ◆ It imparts the Hagedorn temperature to particles in contact with it = perfect thermostat!
- ◆ It is also a perfect particle reservoir!
- ◆ Grand canonical treatment should be used with great care! Microcanonical one is the right one.
- ◆ Our results justify the Statistical Hadronization Model and explain why hadronization T and inverse slopes in el. particle collisions are about 170 MeV.
- ◆ This is phase transition in finite system. No liberation of color d.o.f. is necessary for that!



$A+A$ data

**$h+h, e+e$
data**